Generalized Lagrange's Interpolation Polynomial

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Lets we have n distinct points $x_1,\ x_2,\ x_3,\ \cdots x_n$ on the x-axis. Let more $\varphi_1(x),\ \varphi_2(x),\ \varphi_3(x),\ \cdots \varphi_n(x)$ are n monotonous functions in an interval, containing theses points. We must include in ours assumptions that the products of ours functions are independent.
We are searching for a polynomial with variables $\varphi_i(x)$ of degree, less than, n-1 which passes through n points with coordinates $(x_i, y_i)\ \ i = 1, \cdots, n$.
There is only one polynomial with this property. Let us denote it, the only one of its kind, by $f(x)$.
We substitute

$$W(x) = (\varphi_1(x) - \varphi_1(x_1))\,(\varphi_2(x) - \varphi_2(x_2))\,(\varphi_3(x) - \varphi_3(x_3)) \cdots\ (\varphi_n(x) - \varphi_n(x_n))$$

Then $W(x_i) = 0$ for all x_i.
We denote by W_i the quotient $W_i(x) = \frac{W(x)}{\varphi_i(x) - \varphi_i(x_i)}$. $\quad W_i(x_j) = 0$ if $i \neq j$ and $W_i(x_i) \neq 0$.
Normalizing we get $w_i(x) = \frac{W_i(x)}{W_i(x_i)}$ - n polynomials of φ of degree n-1.

$$w_i(x_j) = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$$

Then $f(x) = y_1 w_1(x) + y_2 w_2(x) + y_3 w_3(x) + \cdots y_n w_n(x)$ is the desired polynomial.
At full its beauty the formula is

$$f(x) = \sum_{i=1}^{n} y_i \frac{(\varphi_1(x) - \varphi_1(x_1))\,(\varphi_2(x) - \varphi_2(x_2)) \cdots (\varphi_{i-1}(x) - \varphi_{i-1}(x_{i-1}))(\varphi_{i+1}(x) - \varphi_{i+1}(x_{i+1})) \cdots\ (\varphi_n(x) - \varphi_n(x_n))}{(\varphi_1(x_i) - \varphi_1(x_1))(\varphi_2(x_i) - \varphi_2(x_2)) \cdots (\varphi_{i-1}(x_i) - \varphi_{i-1}(x_{i-1}))(\varphi_{i+1}(x_i) - \varphi_{i+1}(x_{i+1})) \cdots (\varphi_n(x_i) - \varphi_i(x_n))}$$